## **Problem 1: Perfect Numbers**

Greek mathematicians took a special interest in numbers that are equal to the sum of their proper divisors (a proper divisor of n is any divisor less than n itself). They called such numbers *perfect numbers*.

For example, 6 is a perfect number because it is the sum of 1, 2, and 3, which are the integers less than 6 that divide evenly into 6. Similarly, 28 is a perfect number because it is the sum of 1, 2, 4, 7, and 14.

Write a predicate function **isPerfect(n)** that returns **true** if the integer **n** is perfect, and **false** otherwise.

## **Problem 2: Easter**

Easter falls on the first Sunday after the first full moon following the vernal equinox. Given this definition, the calculation of the date of Easter in a given year involves interacting cycles of the day of the week, the orbit of the moon, and the passage of the sun through the zodiac. Early algorithms for solving this problem date back to the third century and are described in the writings of the eighth-century scholar known as the Venerable Bede. In 1800, the German mathematician Carl Friedrich Gauss published an algorithm for determining the date of Easter that was purely computational in the sense that it relied on arithmetic rather than looking up values in tables. His algorithm—translated from the German—appears below. Although it only works for years in the 18<sup>th</sup> and 19<sup>th</sup> centuries, you can find modifications that work for all years.

FIGURE 4-9 Gauss's algorithm for computing the date of Easter

- I. Divide the number of the year for which one wishes to calculate Easter by 19, by 4, and by 7, and call the remainders of these divisions *a*, *b*, and *c*, respectively. If the division is even, set the remainder to 0; the quotients are not taken into account. Precisely the same is true of the following divisions.
- II. Divide the value 19a + 23 by 30 and call the remainder d.
- III. Finally, divide 2b + 4c + 6d + 3, or 2b + 4c + 6d + 4, choosing the former for years between 1700 and 1799 and the latter for years between 1800 and 1899, by 7 and call the remainder *e*.

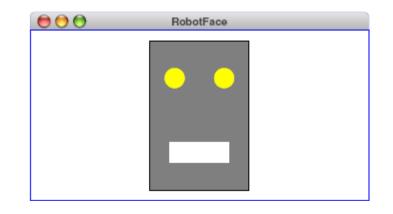
Then Easter falls on March 22 + d + e, or when d + e is greater than 9, on April d + e - 9.

Translated from Karl Friedrich Gauss, "Berechnung des Osterfestes," August 1800 http://gdz.sub.uni-goettingen.de/no\_cache/dms/load/img/?IDDOC=137484

Write a JavaScript function **findEaster(year)** that returns a string showing the date of Easter in the specified year. For example, calling **findEaster(1800)** returns the string "April 13" because that is the date of Easter in the year that Gauss published his algorithm.

## **Problem 3: Drawing a Robot**

Your job is to draw a robot-looking face like the one shown in the following sample run:



This simple face consists of four parts—a head, two eyes, and a mouth—which are arranged as follows:

- *The head*. The head is a big rectangle whose dimensions are given by the named constants **HEAD\_WIDTH** and **HEAD\_HEIGHT**. The interior of the head is gray.
- *The eyes*. The eyes should be circles whose radius in pixels is given by the named constant **EYE\_RADIUS**. The centers of the eyes should be set horizontally a quarter of the width of the head in from either edge and one quarter of the distance down from the top of the head. The eyes are yellow.
- *The mouth*. The mouth should be centered with respect to the head in the *x*-dimension and one quarter of the distance up from the bottom of the head in the *y*-dimension. The dimensions of the mouth are given by the named constants **MOUTH\_WIDTH** and **MOUTH\_HEIGHT**. The mouth is white.

Finally, the robot face should be centered in the graphics window.